10-5
Base e and the Natural Log

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2.71828$$

n	e	
1	2	
10	2.5937	$\left(1+\frac{1}{10}\right)^{10}$
100	2.70481	$\left(1+\frac{1}{100}\right)^{100}$
1000	2.7169	$(1.001)^{1000}$
10,000	2.7181	1.0001^{10000}

$\log_e x = \ln x$

Exponential Form	Logarithmic Form
$e^{2.64} = 14$	ln 14 = 2.64
$e^{3.09} = 22$	ln 22 = 3.09
$e^{-1.1} = 1/3$	$\ln \frac{1}{3} = -1.1$
$e^{1/5} = 1.22$	ln 1.22 = 1/5
$e^4 = 54.6$	$\ln 54.6 = 4$

Simplify:

$\ln e^4 = 4$

$\ln e^2 = 2$

$\ln \frac{1}{e^3} = -3$

Simplify:

$\ln 1 = 0$

$\ln \sqrt{e} = \frac{1}{2}$

$\ln e = 1$

Simplify:

$e^{\ln 17}$ $= 17$

$e^{\ln 21}$ $= 21$

$\ln e^{4x+3}$ $= 4x+3$

Solve

$\ln 3x = 2$

$e^2 = 3x$

$\frac{e^2}{3} = x$

$2.4630 = x$

Solve

$\ln (x-5) = 4$

$e^4 = x - 5$

$e^4 + 5 = x$

$59.5982 = x$

Solve

$\ln 2x + \ln x = \ln 8$

$\ln 2x^2 = \ln 8$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

$\boxed{x = 2}$

Solve

$e^{4x} = 24$

$\ln e^{4x} = \ln 24$

$4x = \ln 24$

$x = \frac{\ln 24}{4} = .7945$

$\log e^{4x} = \log 24$

$4x \log e = \log 24$

$x = \frac{\log 24}{4 \log e}$

If interest is compounded continuously, use the formula:

$A = Pe^{rt}$

A = amount
P = principal
r = rate
t = time

$A = Pe^{rt}$

If $1,000 is compounded continuously at 6% interest:
- How much money would there be in one year?
- How much money would there be in 8 years?

$A = 1000e^{.06(1)}$

$1061.84

$1000e^{.06(8)}$

$= \$1616.07$

How long would it take that same principal to reach at least $1350.

$1350 = 1000e^{.06t}$

$1.35 = e^{.06t}$

$\ln 1.35 = \ln e^{.06t}$

$\ln 1.35 = .06t$

5 yrs

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$3000 = 1000\left(1+\frac{.12}{4}\right)^{4t}$

$3 = (1.03)^{4t}$

$\log 3 = \log 1.03^{4t}$

$\frac{\log 3}{4\log 1.03} = \frac{4t\log 1.03}{4\log 1.03}$

9.3 yrs = t